Math Circles - Intro to Combinatorics - Winter 2024

Problem Set 3

February 21th, 2024

1. We will start with looking at the diagonals of the triangle.

   (a) What do you notice about the first diagonal?

   (b) What do you notice about the second diagonal?

   (c) What do you notice about the third diagonal? Try to find a way to draw these numbers using dots in a pattern.

   (d) Can you think of a way that the pattern in the third diagonal might extend to the fourth and fifth diagonal?

2. Do you notice anything if you mark all the even number?

3. Or the odd numbers?

4. What about number that are divisible by three? Or any other number.

5. Do you notice anything special about the rows that correspond to prime numbers? Does this property work for numbers that are not prime?

6. What happens if you alternate adding and subtracting the entries in a row of pascals triangle?

7. Starting at any number in the pascals triangle, imagine walking through the hexagons. Starting at any number how many different paths can you find to get to the top most one?

8. Imagine you shift the triangle so that it is left justified. This means writing the rows so the first entries all line up, then the second entries, and so on. Your picture should now look like a staircase, with the the top of the stairs on the left. Now take each diagonal and add up the entries. These numbers form a special sequence in math. Do you know it? Write out the sequence and try to find how each number is obtained from the previous numbers.

9. Imagine each row of Pascal's triangle is a number. So we have 1, 11, 121, and so on. What pattern can you find in these numbers?